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(54) IMPROVEMENTS IN OR RELATING TO FOURIER ANALYSERS

(71) We, NATIONAL RESEARCH DEVELOPMENT CORPORATION, a British corporation established by Statute, of Kingsgate House, 66/74 Victoria Street, London, S.W.1., do hereby declare the invention, for which we pray that a patent may be granted to us, and the method by which it is to be performed, to be particularly described in and by the following statement:—

This invention relates to a quadrature spatial frequency Fourier analyser.

Fourier analysers find application, for example, in the fields of radiography and the testing of lenses and other optically transmissive elements or systems, where it is desired to determine the optical transfer function and/or line spread function of a light-transmitting element or system. By using an optical Fourier transformation a non-periodic and discontinuous function may be represented in terms of a continuous function, e.g. as a graphically presented curve. For example, in the field of radiography one can measure the optical transfer function of an X-ray fluorescent screen photoelectrically by an area type masking method which can be applied easily to the fluorescent screen.

The present invention concerns an analyser which, for a sequence of discrete spatial frequencies, can determine the amplitude of the one-dimensional Fourier transform of a finite area of an image, i.e. the image being regarded as a spatial distribution of light intensity.

Known Fourier analysers can be divided into two distinct categories. Firstly, analysers dependent on scanning methods, wherein the optical intensity distribution along the scanned path is converted into an electrical analogue which is then analysed by suitable electronic devices to give a spectrum representative of power per unit bandwidth versus frequency. From a knowledge of the scanning speed this can be equivalently expressed as relative power per unit bandwidth versus spatial frequency.

Such scanning methods present almost insuperable problems when trying to analyse an image which itself has been produced by a scanning technique. For example, noise measurement by photographic integration followed by scanned analysis suffers from limited accuracy, dynamic range and integrating time, due to photographic limitations, largely attributable to the non-linearity of the process.

The second category of analysers are those using non-scanning methods, which have however so far been restricted to analysing line spread functions. In such methods the object is imaged on to a mask whose effective transmission varies sinusoidally in one direction. When a maximum of mask transmission coincides with the point of maximum intensity of the spread function, the light flux passing through the mask will be a maximum and will yield the cosine Fourier transform amplitude at the spatial frequency of the mask. If the mask is displaced through 90°, the light flux will give the sine transform amplitude. For a symmetrical spread function the sine transform vanishes. The phase angle relative to the maximum intensity can be obtained as a function of spatial frequency from the ratio of the two transforms, and the curve representing the amplitude of the power versus spatial frequency can be found for each frequency from the quadratic sum of the two transforms. This method cannot however be applied to a dynamic image whose intensity distribution varies in time, because the phases of the components cannot be identified.

The analyser of the present invention on the other hand differs from analysers of said second category in that it can measure phase relative to the mask position, which is assumed to be fixed relative to some fixed origin, whereas such known analysers determine phase rela-

tive to the line spread maximum intensity. Hence the analyser of the present invention can determine the Modulus of the Fourier Coefficient at a given spatial frequency regardless of the phase relationship between object and mask intensity distributions.

In accordance with the present invention, there is provided a Fourier analyser for measuring the one-dimensional spatial frequency spectrum of an image comprising optical means producing multiple images of the intensity distribution of said image under analysis, filter means having transmission characteristics varying sinusoidally in one direction and arranged to provide simultaneously from said multiple images outputs indicative of the sine and cosine transforms of the image under analysis, and means arranged to quadratically sum said outputs and provide a unidirectional signal representative of the spectrum amplitude at the spatial frequency of said filter means.

A feature of the present invention resides in the ability of the analyser to deal with images whose intensity distribution varies temporally as well as spatially. This means, in particular, that it can analyse noise distributions and images generated by scanning methods.

The analyser of the present invention has a number of unique facilities: it will give, for a range of discrete spatial frequencies, the amplitude of the Fourier transform of

a) images whose Fourier components are of unknown phase;
b) dynamic images, typically produced by a quantum-limited scintillation process; and,
c) scanned images, which may or may not additionally belong in b).

These potentialities result from the analyser including filters to give simultaneously the values of the sine and cosine transforms of an image and a means of simulating negative transmission, thus enabling the analyser to make band-pass rather than low-pass measurements, as well as generating a D.C. output signal, leading to simple processing methods and making quasi-infinite integration possible.

In one embodiment of analyser according to the present invention, the object is simultaneously imaged on to two sinusoidal masks of relative phase displacement $\pi/2$, the quadratic sum of the outputs then giving the spectrum amplitude at the spatial frequency of the mask. The spatial frequency component being measured can be visualised as a unidimensional sinusoidal variation in intensity of the same spatial frequency as the mask, so that for a relative mask-image phase difference of zero the intensity and transmission maxima coincide. However, the value of the quadratic sum is independent of the relative phase angle, which consequently need not be known.

In other words, the spatial frequency Fourier transform can be written

$$F(s)=\int a(w)\cos(ws)\,dw+\int b(w)\sin(ws)dw \quad (1)$$
$$=\int S(w)\cos(ws+\phi)\,dw \quad (2)$$

where

F(s) is the spatial intensity distribution;
W is the angular spatial frequency;
a(w) is the cosine Fourier transform amplitude;
b(w) is the sine Fourier transform amplitude;
S(w) is the modulus of the complex angular spatial frequency spectrum; and
ϕ(w) is the relative phase angle at frequency W.

The main output required from the analyser is a plot of S(w) versus W.

From (1) and (2):

$$S(w)^2=a(w)^2+b(w)^2; \quad (3)$$

and,

$$\phi(w)=\tan^{-1}\left[\frac{-b(w)}{a(w)}\right] \quad (4)$$

If the mask transmission maximum and spatial frequency component amplitude maximum do not coincide but differ by an angle δ, this is equivalent to adding a constant error angle δ in equations (1) and (2), which become;

$$F(s)=\int a(w)\cos(ws+\delta)dw+\int b(w)\sin(ws+\delta)dw \quad (1a)$$
$$=\int S(w)\cos(ws+\delta+\phi)dw. \quad (2a)$$

Equation (3) is unchanged under these circumstances, as can be seen by inspection.

Equation (4) becomes

$$\phi(w)+\delta=\tan^{-1}\left[\frac{-b(w)}{a(w)}\right]. \quad (4a)$$

Thus the sole result is that the reference angle, relative to which ϕ(w) is measured, has been changed by δ. Although the reference is quite arbitrary, and the only interest in phase concerns phase change with frequency and never absolute phase, the relative shift ϕ(w) cannot be measured unless the error δ can be separated from the sum $\delta+\phi$(w). For a dynamic image this will involve some form of instan-

taneous servomechanism to continually adjust the spatial position (phase angle) to maximise the output from the cosine transform filter. For a static image such separation could be attempted, the phase shift required to maximise the cosine output being δ.

The mask can have a band-pass property only if both positive and negative transmission is possible. This can be simulated for example by subtracting from the orthogonal transform signals a mean level signal, also derived from the object.

The electrical output is unidirectional:

1) For an unscanned static image the output is a steady current;

2) for an unscanned dynamic image the output is a varying current. The total charge integrated electrically in a given time of observation is the same as would result if the dynamic image could be optically integrated for the same time and the integral presented to the mask. Thus a difficult technical problem in image storage is replaced by a simple electrical integration;

3) for a scanned image the output is a varying current whose integral over one scan period gives the same charge as would be unscanned original image if displayed for one scan duration.

The form and positioning of the optical components and the form of the filters used may take several alternative forms. The accompanying drawing illustrates one embodiment of analytical apparatus in accordance with the invention.

As shown in the drawing, the analyser basically comprises a mirror tunnel formed by two parallel mirrors M1 and M2. An original image OI having an intensity distribution to be analysed is arranged to be produced at one end of the tunnel and an objective lens L is positioned adjacent to the other end of the tunnel. Virtual images VI of the intensity distribution to be analysed are formed by the parallel mirrors M1 and M2 and three images are thereby formed in the plane of three spaced masks S1, S2 and S3, which may for example be formed on a single photographic plate. Behind each mask is a photomultiplier P1, P2, P3 or other linear photosensitive device providing outputs A, B and C respectively.

Masks S1 and S3 are of sinusoidally varying transmission in one direction and are displaced through $\pi/2$ relative to each other. Mask S2 has a transmission characteristic which enables the mean luminance to be determined.

The orthogonal transform amplitudes are obtained from difference circuits 10 and 11 by subtracting the mean output B of photomultiplier P2 from the outputs A and C of photomultipliers P1 and P3 respectively. The orthoganol transform amplitudes are then quadratically added by suitable circuitry 12, for example, comprising squaring circuits and a summing amplifier.

The optical duplication which is provided by the mirror tunnel M1, M2 may alternatively be provided by using a prismatic beam splitter, or a large collimating lens and a number of smaller imaging lenses, for example.

In an alternative embodiment four masks may be used, one pair containing alternate half cycles of a sine wave. Behind each mask of this pair a photomultiplier is again used to provide an output signal and the difference in electrical outputs gives the sine transform amplitude. A corresponding arrangement for the other pair of masks gives the cosine transform amplitude.

In either of these embodiments the masks may be either of variable density type or of variable area type.

Variable density masks vary sinusoidally in transmission in one direction and have constant transmission at right angles to it.

Variable area masks consist of an opaque background with a sinusoidal variation in transparent area in one direction. If a cylindrical optic is added to a variable area mask in the imaging path to defocus the image at right angles to the direction of mask variation, the mask will have the same effect on the light flux transmitted as in a variable density mask.

In general, the analyser system can measure the one-dimensional spatial frequency spectrum in any image, including those formed by a scanning process.

For a stationary image no temporal problems exist but for an image which is quantum limited the integrated intensity distribution will be dependent on the build-up time allowed. In such cases photographic integration might seem the obvious method but the number of processing techniques involved, each capable of causing a considerable change in the recorded or reproduced image, and the non-linearity of the photographic process and its limited dynamic range makes this of limited use. As long as the quantised imaging process is truly stationary in time, so that sequential measurements each for the same statistically long time will yield the same results, the apparatus hereinbefore described will perform a transform analysis. In principle, by increased optical complexity, measurements throughout the spectrum could be made simultaneously, but this would be a unique measure of the spectrum during that time only and would have no general applicability unless the process were stationary in time, in which case the much simpler sequential method is just as valid.

At any instant the image has a one dimensional Fourier transform whose amplitude varies with frequency. If the quadratic addition of sine and cosine components is performed as a continuous process, a continuous

record of the variation with time of transform amplitude is obtained. However, in general, one seeks the true or "infinite time" spectrum, which can be obtained as the average of the vector indicated at any instant in the continuous record or much more simply by integrating the sine and cosine signals for the required period and then quadratically adding.

For truly random noise the average value of the sine and cosine transforms will be identical, and so the phase difference obtained from a given ratio of integrated sine and cosine signals can be used as a measure of spatial randomness or lack of correlation in the image. The sensitivity of the test for randomness will depend on the inherent signal-to-noise ratio of the measuring apparatus.

For the measurement of spectra using long integration times the technique is basically D.C. pulse integration and involves no high-frequency analogue circuits. For a continuous record, analogue computation at frequencies as high as 20Mc/s is possible.

The spatial frequency range over which the apparatus will function is determined at low frequencies by the bandwidth of the filtering process (e.g. size of mask) and at high frequencies by the quality of optics obtainable, the geometrical and mechanical design chosen, and by the quality of the sinusoidal masks or grids.

Within limits set by the physical design of the apparatus the bandwidth of measurement can be shaped as required, the actual bandwidth being reciprocally related to the length of the sinusoidal mask or grid and the shape determined by the transform of the envelope of the filter sinusoid, e.g. $\frac{\sin x}{x}$ for the rectangular envelope of a simple truncated sinusoid. By suitable choice of optical reduction or magnification any required frequency range can be obtained.

WHAT WE CLAIM IS:—

1. A Fourier analyser for measuring the one-dimensional spatial frequency spectrum of an image comprising optical means producing multiple images of the intensity distribution of said image under analysis, filter means having transmission characteristics varying sinusoidally in one direction and arranged to provide simultaneously from said multiple images outputs indicative of the sine and cosine transforms of the image under analysis, and means arranged to quadratically sum said outputs and provide a unidirectional signal representative of the spectrum amplitude at the spatial frequency of said filter means.

2. A Fourier analyser as claimed in claim 1, wherein said filter means act as band-pass filters.

3. A Fourier analyser as claimed in claim 1 or 2, wherein said filter means comprise variable density masks.

4. A Fourier analyser as claimed in claim 1 or 2, wherein said filter means comprise variable area masks.

5. A Fourier analyser as claimed in any preceding claim, wherein said optical means provides three images of the original, wherein said filter means comprises three masks, two of which are of sinusoidally varying transmission in one direction and are displaced through 90° relative to each other and the third of which has a transmission characteristic enabling the mean luminance to be determined, wherein linear photosensitive devices are positioned at the output side of each mask, and wherein the orthogonal transform amplitudes are obtained by subtracting the mean output of said third mask from the outputs of the respective other two masks.

6. A Fourier analyser as claimed in any of claims 1 to 4, wherein said optical means provides four images of the original, and wherein said filter means comprises four masks, the sine transform amplitude being obtained from the difference in the outputs of two of said masks, and the cosine transform amplitude being obtained from the difference in the outputs of the other two masks.

7. A Fourier analyser as claimed in any of claims 1 to 5, wherein said optical means comprises a mirror tunnel formed by two parallel mirrors having the image under analysis located therebetween.

8. A Fourier analyser as claimed in any of claims 1 to 6, wherein said optical means comprises a prismatic beam splitter.

9. A Fourier analyser as claimed in any preceding claim when used for analysing dynamic images, which includes a servomechanism arranged to continually maximise the output of the cosine transform filter means.

10. A Fourier analyser substantially as hereinbefore described with reference to the accompanying drawings.

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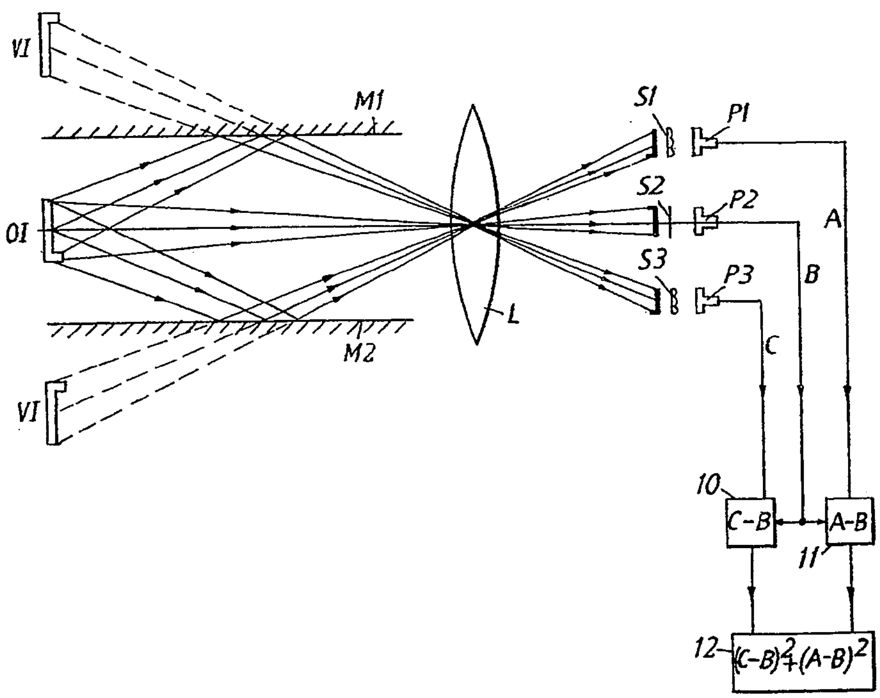
VI
M1
S1
P1
S2
P2
A
OI
S3
P3
B
L
C
M2
VI
10
C-B
A-B
11
12
(C-B)²+(A-B)²